UMN-TH-1318/94
hep-ph/yymmddd
November 1994

Nucleosynthesis and the Time Dependence of Fundamental Couplings

Bruce A. Campbell
Department of Physics, University of Alberta
Edmonton, Alberta, Canada T6G 2J1
and
Keith A. Olive
School of Physics and Astronomy, University of Minnesota
Minneapolis, MN 55455, USA

Abstract

We consider the effects of the time dependence of couplings due to their dependence on a dilaton field, as occurs in superstring theory, as well as in gravity theories of the Jordan-Brans-Dicke type. Because the scale parameters of couplings set by dimensional transmutation depend exponentially on the dilaton vev, we may obtain stringent limits on the shift of the dilaton from the requirement that the induced shift in the couplings not vitiate the successful calculations of element abundances for big-bang nucleosynthesis. These limits can be substantially stronger than those obtained directly from the dilaton-induced change in the gravitational coupling.

The successful predictions of the light element abundances in the standard model of big bang nucleosynthesis (SBBN) [1] provides a basis to test extensions to the standard model of particle interactions. While deviations to SBBN typically induce changes in all of the light element abundance predictions (D, ^{3}He, ^{4}He, and ^{7}Li), particle physics models are mostly constrained by the ^{4}He mass fraction, Y_p. In the SBBN, the abundances are primarily sensitive to only a single parameter, the baryon-to-photon ratio, η. Consistency between the predictions of SBBN and the observational determinations of the light element abundances restricts η to a narrow range between 2.8×10^{-10} and 4×10^{-10}. In this range the calculated ^{4}He mass fraction lies in the range $Y_p = 0.239-0.246$ [2]. This is slightly high when compared with the observationally inferred best primordial value, $Y_p = 0.232 \pm 0.003 \pm 0.005$ [3], where the errors are 1 σ statistical and systematic errors respectively. Indeed, consistency relies on these errors and even so, allows for very little breathing room for any enhancement in primordial ^{4}He. This is the reason that one can obtain very tight limits on the number of neutrino flavors [4, 3, 5].

The ^{4}He abundance is primarily determined by the neutron-to-proton ratio just prior to nucleosynthesis which before the freeze-out of the weak interaction rates at a temperature $T_f \sim 1$ MeV, is given approximately by the equilibrium condition

$$(n/p) \approx e^{-\Delta m_N/T_f} \tag{1}$$

where $\Delta m_N = 1.29$ MeV is the neutron-proton mass difference. (The ratio is slightly altered by free neutron decays between T_f and the onset of nucleosynthesis at about $T \sim 0.1$ MeV.) Furthermore, freeze-out is determined by the competition between the weak interaction rates and the expansion rate of the Universe

$${G_F}^2 {T_f}^5 \sim \Gamma_{\rm wk}(T_f) = H(T_f) \sim \sqrt{G_N N} {T_f}^2 \tag{2}$$

where N counts the total number of relativistic particle species. The presence of additional neutrino flavors (or any other relativistic species) at the time of nucleosynthesis increases the overall energy density of the Universe and hence the expansion rate leading to a larger value of T_f, (n/p), and ultimately Y_p. Because of the form of eq. (2) it is clear that just as

one can place limits on N, any changes in the weak or gravitational coupling constants can be similarly constrained [6]-[11].

Constraints on G_N and G_F have often been obtained under the assumption that these quantities have varied in time as a power-law, $G \propto t^x$. Constraints on $\delta G/G$ yield an acceptable range for x [6, 11]. Limits on these couplings as well as the fine structure constant and neutron-proton mass difference were considered in [7]. In general, Y_p is most sensitive to changes in Δm_N [7]. It was pointed out in [9] however, that as the Fermi constant can be written directly as the vev of the Higgs boson in the standard model, $G_F/\sqrt{2} = 1/2v^2$, changes in G_F will naturally induce changes in fermion masses and hence Δm_N. In this context, temporal as well as spatial changes in G_F were considered in [10].

In string theory, the vev of the dilaton field, acts as the string loop counting parameter [12]. At the (string) tree level, changes in the vacuum value of the dilaton corresponds directly to changes in the gravitational coupling G_N. This can be seen by writing down the action in the string frame [13],

$$\begin{aligned} S &= \int d^4x\sqrt{g}e^{-\sqrt{2}\kappa\phi}\left(\frac{1}{2\kappa^2}R + \partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu y\partial^\mu y - m_y^2 y^2 \right. \\ & \left. -\bar{\psi}\gamma^\mu D_\mu\psi - m_\psi\bar{\psi}\psi - \frac{\alpha'}{16\kappa^2}F_{\mu\nu}F^{\mu\nu}\right) \end{aligned} \tag{3}$$

where ϕ is the dilaton field, y is an arbitrary scalar field and ψ is an arbitrary fermion. D_μ is the gauge-covariant derivative corresponding to gauge fields with field strength $F_{\mu\nu}$. $\kappa^2 = 8\pi G_N$. However, by performing a conformal transformation, $g_{\mu\nu} \to e^{-\sqrt{2}\kappa\phi}g_{\mu\nu}$ we can rewrite (3) in the Einstein frame as

$$\begin{aligned} S &= \int d^4x\sqrt{g}\left(\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu y\partial^\mu y - e^{\sqrt{2}\kappa\phi}m_y^2 y^2 \right. \\ & \left. -\bar{\psi}\gamma^\mu D_\mu\psi - e^{\kappa\phi/\sqrt{2}}m_\psi\bar{\psi}\psi - \frac{\alpha'}{16\kappa^2}e^{-\sqrt{2}\kappa\phi}F_{\mu\nu}F^{\mu\nu}\right) \end{aligned} \tag{4}$$

Now it apparent that changes in the dilaton vev, will induce changes in gauge couplings and fermion masses (see details below). Thus, we will be able to limit changes in the dilaton vev from the time of nucleosynthesis to today from the observed ^{4}He abundance. As we

will see, we will be able to obtain particularly stringent limits because the dependence on ϕ of gauge and Yukawa couplings induce changes in quantities such as the Higgs vev, v, and Λ_{QCD} which are exponentially dependent on the dilaton vev through the renormalization group equations and dimensional transmutation. In what follows, we will first derive the general (though approximate) relations between Y_p and the various couplings and Δm_N and the corresponding limits on these quantities. We will then derive the induced changes in these quantities from changes in the dilaton vev and hence derive limits on the changes of the dilaton vev in the context of string theory. We will also consider these same effects in the context of Jordan-Brans-Dicke gravity.

As is well known, the ^{4}He abundance is predominantly determined by the neutron-to-proton ratio and is easily estimated assuming that all neutrons are incorporated into ^{4}He,

$$Y_p \approx \frac{2(n/p)}{1+(n/p)} \tag{5}$$

so that

$$\frac{\Delta Y}{Y} \approx \frac{1}{1+(n/p)} \frac{\Delta(n/p)}{(n/p)} \tag{6}$$

From eqs.(1) and (2) it is clear that changes in any of the quantities G_F, G_N, or N, will lead to a change in T_f and hence Y_p. If we keep track of the changes in T_f and Δm_N separately, we can write,

$$\frac{\Delta(n/p)}{(n/p)} \approx \frac{\Delta m_N}{T_f} \left(\frac{\Delta T_f}{T_f} - \frac{\Delta^2 m_N}{\Delta m_N} \right) \tag{7}$$

where $\Delta^2 m_N$ is the change in Δm_N. Combining these equations we obtain

$$\frac{\Delta Y}{Y} \approx \left(\frac{\Delta T_f}{T_f} - \frac{\Delta^2 m_N}{\Delta m_N} \right) \tag{8}$$

where the factor $\Delta m_N / T_f(1+(n/p)) \approx 1$. From the consistency of the light elements, we will take the limit $-0.08 < \Delta Y/Y < 0.01$ assuming a SBBN value of $Y_p = 0.240$ and the observed range to be $0.221 < Y_p < 0.243$.

As noted above, changes in T_f are induced by changes in the weak and gravitational couplings and can be readily determined from (2). Changes in Δm_N can come from a

number of sources. One can write the nucleon mass difference as

$$\Delta m_N \sim a\alpha_{em}\Lambda_{QCD} + bv \tag{9}$$

where a and b are dimensionless constants giving the relative contributions from the electromagnetic and weak interactions. In (9), v is the standard model Higgs expectation value. A discussion on the contributions to Δm_N can be found in [14]. We will take $a \simeq -.8\text{MeV}/\alpha_{em0}\Lambda_{QCD_0}$ where Λ_{QCD_0} is the present (low energy value) of Λ_{QCD}, $\alpha_{em0}^{-1} \simeq 137$ and $b \simeq 2.1\text{MeV}/v_0$ where v_0 is the standard value of the Higgs expectation value, $v_0 \simeq 247$ GeV. Our results will not be particularly sensitive to the precise values of a and b.

In what follows we will consider the effects of changes in gauge and Yukawa coupling constants. From eq. (9) we see that a change in the electromagnetic coupling constant will directly induce a change in Δm_N. Changes in the strong coupling constant however can be seen to have dramatic consequences from the running of the renormalization group equations[1]. Indeed the QCD scale Λ is determined by dimensional transmutation

$$\alpha_3(M_P^2) \equiv \frac{g_3^2(M_P^2)}{4\pi} \approx \frac{12\pi}{(33-2N_f)\ln(M_P^2/\Lambda^2)} \tag{10}$$

or for $N_f = 3$

$$\Lambda^2 = M_P^2 \exp\left(\frac{-48\pi^2}{27 g_3^2(M_P^2)}\right) \tag{11}$$

Clearly, changes in g_3 will induce (exponentially) large changes in Λ_{QCD} and therefore in Δm_N and Y_p.

Similarly, changes in Yukawa couplings can induce large changes in Δm_N. In models in which the electroweak symmetry is broken radiatively, the weak scale is also determined by dimensional transmutation [15]. This mechanism is based on the solution for the renormalization scale at which the Higgs mass^2 goes negative, being driven by a Yukawa coupling, presumably h_t. The weak scale and the Higgs expectation value then corresponds to the renormalization point and is given qualitatively by

$$v \sim M_P \exp(-2\pi c/\alpha_t) \tag{12}$$

[1]This observation was made by Dixit and Sher [9] in their criticism of the dependence of Y_p on α_3 in [8]

where c is a constant of order 1, and $\alpha_t = h_t^2/4\pi$. Thus small changes in h_t will induce large changes in v and hence in T_f and Δm_N.

Let us now look at the implications of a rolling dilaton in string theory. We will work in the Einstein frame and therefore, we will not consider changes in the gravitational coupling G_N. From the form of the action in (4) one can see that although scalars and fermions have canonical kinetic terms after the conformal transformation, there remains a dilaton dependence in their masses as well as in the coefficient of the gauge field strength. From (4), we define the gauge coupling constant

$$\frac{1}{g^2(M_P^2)} = \frac{\alpha' e^{-\sqrt{2}\kappa\phi}}{2\kappa^2} = \frac{\alpha' S_R}{2\kappa} \tag{13}$$

S is the (chiral) multiplet in which the real part of the scalar is associated with the dilaton and is used here for convenience. α' is the string tension. From (11), we see that Λ_{QCD} is in fact doubly exponentially dependent on the dilaton[2] ϕ

$$\Lambda = M_P \exp\left(\frac{-12\pi^2\alpha' S_R}{27\kappa}\right) \tag{14}$$

Therefore we can write the induced change in Δm_N as

$$\frac{\Delta^2 m_N}{\Delta m_N} = \frac{a\alpha\Lambda}{\Delta m_N}\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda}\right) \simeq \frac{-.8}{1.3}\left(-\frac{\Delta S_R}{S_R} - \frac{12\pi^2\alpha'}{27\kappa}\Delta S_R\right) \tag{15}$$

Similarly, we see from (4) that Yukawa couplings are also expected to be dilaton dependent. If we assume that fermion masses are generated by the Higgs mechanism, then the corresponding Yukawa term in the Lagrangian would be

$$h e^{\kappa\phi/\sqrt{2}} H \bar{\psi}\psi \tag{16}$$

and the ψ mass is given by $m_\psi = h e^{\kappa\phi/\sqrt{2}}\langle H\rangle = h e^{\kappa\phi/\sqrt{2}} v/\sqrt{2}$. The effective Yukawa coupling is therefore given by $h e^{\kappa\phi/\sqrt{2}}$. Now, as a consequence of eq. (12), the Higgs vev is determined from

$$v = M_P \exp\left(\frac{8\pi^2 c\kappa S_R}{h_t^2}\right) \tag{17}$$

[2]The dependence of Λ_{QCD} on the dilaton was utilized in a discussion regarding the dilaton coupling to matter [16] and to our notion of the expansion of the Universe in [17]

In this case, the induced change in Δm_N is

$$\frac{\Delta^2 m_N}{\Delta m_N} = \frac{bhM_P}{\Delta m_N}\frac{d}{dS_R}\left(\frac{1}{\sqrt{S_R}}\exp(\frac{8\pi^2 c\kappa S_R}{h_t^2})\right)\Delta S_R \simeq -\frac{2.1}{1.3}\left(\frac{1}{2}+\frac{8\pi^2 c\kappa S_R}{h_t^2}\right)\frac{\Delta S_R}{S_R} \qquad (18)$$

Note that in (9) it is really the quark mass difference which contributes to Δm_N so that the dependence is hv rather than simply v. In addition, changes in v will also induce changes in the freeze-out temperature T_f,

$$\frac{\Delta T_f}{T_f} = \frac{4}{3}\frac{\Delta v}{v} = \frac{-32\pi^2 c\kappa\Delta S_R}{3h_t^2} \qquad (19)$$

Clearly even small changes in the dilaton expectation value will have dramatic consequences on the ^{4}He abundance.

The contributions in eqs. (15,18 and 19) can all be summed to give a net change in Y_p. For $c \sim h_t \sim \kappa S_R \sim 1$ and $g^2 \sim 0.1$, we have from (8)

$$\frac{\Delta Y}{Y} \sim 200\kappa\Delta S_R \qquad (20)$$

which in order to be consistent with SBBN gives

$$-4\times 10^{-4} \lesssim \kappa\Delta S_R \lesssim 5\times 10^{-5} \qquad (21)$$

The corresponding limits on other quantities can be easily obtained from (21) using relations such as $\Delta G_F/G_F \sim 150\kappa\Delta S_R$, $\Delta\alpha_{em}/\alpha_{em} = -\Delta S_R/S_R$, $\Delta h/h = -\Delta S_R/2S_R$. This is our main result.

Before concluding, it is interesting to consider these same limits in the context of Jordan-Brans-Dicke gravity. If we write down the analogous action to eq. (3)

$$\begin{aligned} S &= \int d^4x\sqrt{g}\left(\phi R - \frac{\omega}{\phi}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu y\partial^\mu y - m_y^2 y^2 \right. \\ & \left. -\bar\psi\gamma^\mu D_\mu\psi - m_\psi\bar\psi\psi - \frac{1}{8g^2}F_{\mu\nu}F^{\mu\nu}\right) \end{aligned} \qquad (22)$$

and perform the analogous conformal transformation, $g_{\mu\nu} \to 2\kappa^2\phi g_{\mu\nu}$, we have

$$
\begin{aligned}
S &= \int d^4x\sqrt{g}\left(\frac{1}{2\kappa^2}R - \frac{(\omega+\frac{3}{2})}{2\kappa^2\phi^2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4\kappa^2\phi}\partial_\mu y\partial^\mu y - \frac{1}{4\kappa^4\phi^2}m_y^2y^2\right. \\
&\left. -\frac{1}{2\kappa^2\phi}\bar{\psi}\gamma^\mu D_\mu\psi - \frac{1}{(2\kappa^2\phi)^{3/2}}m_\psi\bar{\psi}\psi - \frac{1}{8g^2}F_{\mu\nu}F^{\mu\nu}\right)
\end{aligned}
\tag{23}
$$

Notice first, that the coefficient of $F_{\mu\nu}F^{\mu\nu}$ is independent of ϕ and therefore we do not expect that gauge couplings will vary. This is a result of the conformal invarience of the gauge kinetic term. Furthermore, notice also that the fermion and boson kinetic terms are no longer canonical (as well as that of ϕ). If we rescale fermions and bosons by $(\psi, y) \to \sqrt{2\kappa^2\phi}\ (\psi, y)$ and we assume that masses are generated by the expectation value of a scalar (through interactions of the form H^2y^2 and $H\bar{\psi}\psi$ and H is similarly rescaled) then we see that the ϕ dependence of the masses drop out. Therefore, we do not expect effects based on transdimensional mutation as neither gauge nor Yukawa couplings will depend on ϕ to induce changes in Λ_{QCD} and v. The Higgs expectation value will probably still depend on ϕ if its value is determined from a Higgs potential of the form $V = \lambda H^4 - m^2H^2$. If in the standard model $v^2 \sim m^2/\lambda$, then in the conformally transformed JBD theory, $v^2 \sim m^2/\lambda(2\kappa^2\phi)$. Thus $G_F \propto v^{-2} \propto \phi$ and $\Delta G_F/G_F = \Delta\phi/\phi$. This could be put in the form of a constraint on the JBD parameter ω, but will yield constraints which have been discussed recently in the literature [18] and will not be repeated here.

In summary, we have derived limits on any possible time variation (from the time of nucleosynthesis to the present) in the dilaton expectation value (in the context of string gravity) due to its effect on standard model parameters such as gauge and Yukawa couplings as well as Λ_{QCD} and the Higgs expectation value from big bang nucleosynthesis. The induced variation in the latter two quantities (noting that their scales are generated through dimensional transmutation) provides us with stringent limits on ΔS_R which can be translated into limits on other couplings. In the JBD theory of gravity, effects as large were not found as the JBD scalar does not automatically alter gauge and Yukawa couplings.

Acknowledgements

We would like to thank R. Madden for comments on the manuscript. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada and by DOE grant DE-FG02-94ER40823.

References

[1] T.P. Walker, G. Steigman, D.N. Schramm, K.A. Olive and K. Kang, Ap. J. **376** (1991) 51.

[2] P. Kernan, G. Steigman, G. & Walker, T.P. 1994, (in preparation).

[3] K.A. Olive and G. Steigman, Ap.J. Supp. (1995) in press.

[4] G. Steigman, D.N. Schramm, and J. Gunn, Phys. Lett. **B66** (1977) 202.

[5] P. Kernan and L.M. Krauss, Phys. Rev. Lett. **72** (1994) 3309.

[6] J. Yang, G. Steigman, D.N. Schramm, and R.T. Rood, Ap.J. **227** (1979) 697.

[7] E.W. Kolb, M.J. Perry, and T.P. Walker, Phys. Rev. **D33** (1986) 869.

[8] J.D. Barrow, Phys. Rev. **D35** (1987) 1805.

[9] V.V. Dixit and M. Sher, Phys. Rev. **D37** (1988) 1097.

[10] R.J. Scherrer and D.N. Spergel, Phys. Rev. **D47** (1993) 4774.

[11] F.S. Accetta, L.M. Krauss, and P. Romanelli, Phys. Lett. **B248** (1990) 146.

[12] M. Green, J. Schwarz, and E. Witten, *Superstring Theory*, Cambridge University Press (1987); D. Lust and S. Theisen, *Lectures on String Theory* Springer-Verlag (1990); E. Witten, Phys. Lett. **B149** (1984) 351.

[13] C. Lovelace, Phys. Lett. **B135** (1984) 75; E.S. Fradkin and A.A. Tseytlin, Phys. Lett. **B158** (1985) 316; C.G. Callan, D. Friedan, E.J. Martinec and M.J. Perry, Nucl. Phys. **B262** (1985) 593; A. Sen, Phys. Rev. **D32** (1985) 2102; Phys. Rev. Lett. **55** (1985) 1846; C.G. Callan, I.R. Klebanov and M.J. Perry, Nucl. Phys. **B278** (1986) 78.

[14] J. Gasser and H. Leutwyler, Phys. Rep. **87** (1982) 77.

[15] K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, Prog. Theor. Phys. **68** (1982) 927; L.E. Ibáñez and G.G. Ross, Phys. Lett. **B110** (1982) 215; J. Ellis, L.E. Ibáñez and G.G. Ross, Phys. Lett. **B113** (1982) 283; L. Alvarez-Gaumé, M. Claudson, and M. Wise, Nucl. Phys. **B207** (1982) 96; J. Ellis, J.S. Hagelin, D.V. Nanopoulos, and K. Tamvakis, Phys. Lett. **B125** (1983) 275.

[16] J. Ellis, S. Kalara, K.A. Olive and C. Wetterich, Phys. Lett. **B228** (1989) 264.

[17] B.A. Campbell, A.D. Linde, and K.A. Olive, Nucl.Phys. **B355** (1991) 146.

[18] J.A. Casas, J. Garcia-Bellido, M. Quiros, Mod. Phys. Lett. **A7** (1992) 447; Class. Quant. Grav. **9** (1992) 1371; A. Serna, R. Dominquez-Tenreiro and G. Yepes, Ap.J. **391** (1992) 433.